**CIS 606 Spring 2022 SRIHARINI KAZA**

**Homework 1 2801156**

**Question 1:**

Prove the following summation by Mathematical Induction:

=

**Solution:**

Initially, let us consider the base case:

**Base case:**

* Substituting in = = 0 True

**Hypothesis:**

Let us consider the given statement is true for n=p

Using, method of Mathematical Induction for n=p+1

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**Verification:**

Resubstituting (p+1) =n

Therefore, our assumption is True.

There is it proved by Mathematical Induction that:

=

Hence Proved

**Question 2:**

Find a simple formula for

**Solution:**

Given,

=

=

=

Therefore, the simple formula is:

**Question 3:**

Prove that by using the definition of Big -Oh notation.

**Solution:**

We know that f(n) = Og(n) if

f(n) cg(n) ∀ nand for any positive constants c

(+…. O)

When Cp max (C1, C2, C3…Cn)

=

Therefore,

-using Big-Oh notation

Hence Proved

**Question 5:**

In how many ways can n professors sit around a circular conference table? Consider two seating to be the same if one can be rotated to form the other.

**Solution:**

We know that the number of ways for n people sitting in the circular arrangement is (n-1)!

Since,

The seating can be done in: (n-1) (n-2) …. (1) ways

Therefore,

If n professors sit around a circular conference table, considering two seating to be the same if one can be rotated to form the other,

The number of ways they can sit is: **(n-1)!**

So, the solution is (n-1)! ways.

**Question 6:**

In how many ways can we choose three distinct numbers from the set {1, 2, …, 99} so that their sum is even?

**Solution:**

We know that,

n=

Let us consider:

sum of three numbers is even when

1) all 3 numbers are even

2) 2 odd numbers and 1 even number

Even numbers in the set = 49 numbers

Odd numbers in the set = 50 numbers

No. of ways of choosing 3 numbers from 49, if the order of numbers does not matter = 49C3 = 18424

No. of ways of choosing 1 number from 49 even numbers, if the order of numbers does not matter = 49C1 = 49

No. of ways of choosing 2 number from 50 odd numbers if the order of numbers does not matter = 50C2 = 1225

Therefore,

On combining,

1) = 18424

2) = 49\*1225 = 60025

so final answer = 18424+ 60025 = 78449

**→78449 ways**

Thus,

78449 ways can be chosen such that three distinct numbers from the set {1, 2, …, 99} so that their sum is even.

**Question 7:**

Professor Wang flips a fair coin once. Professor Hizlan flips a fair coin twice. What is the probability that Professor Wang obtains more heads than Professor Hizlan?

**Solution:**

If Professor wang toss a coin:

there are only two possibilities = 1 Head or 1 Tail

chance of tossing Head = 1/2 = 0.5

Professor wang has 2 events = H or T

Professor Hizlan has 4 events = HH, HT, TH, TT

Total possible Events = 2\*4 = 8

The number of events where Professor Wang can win is:

1) Professor Wang got 1 head and Professor Hizlan got either TT

so, the probability = no of winning events / total number of events

= **1/8**

Therefore,

The probability that Professor Wang obtains more heads than Professor Hizlan is 1/8

**Question 8:**

Suppose we shuffle a deck of 10 cards, each bearing a distinct number from 1 to 10, to mix the cards thoroughly. We then remove three cards, one at a time, from the deck. What is the probability that we select the three cards in sorted (increasing) order?

**Solution:**

We can analyze the problem in simple terms,

the no of ways of arranging 3 numbers is 3! = 6 ways

so, for every 3 numbers picked one by one randomly, there is always only 1 way where they can be in ascending order since all the numbers are distinct.

so, for every such 3 number picks, there is 1 way

where there is total 6 ways of arranging them

so, the probability = **1/6**

Therefore,

The probability that we select the three cards in sorted order is 1/6.